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Mass, energy, and momentum in Einstein’s mechanics

Newtonian mechanics provides a perfect description of the motion of a body when its velocity $v$ is much less than the velocity of light: $v \ll c$. But this theory is manifestly incorrect when the velocity is comparable with $c$, especially when $v = c$. To describe motion with arbitrary velocity, up to that of light, we have to turn to Einstein’s special theory of relativity, i.e., relativistic mechanics. Newton’s non-relativistic mechanics is but a particular (although very important) limiting case of Einstein’s relativistic mechanics.

The word “relativity” originates in Galileo’s relativity principle. In one of his books, Galileo explained in very graphic terms that no mechanical experiment performed inside a ship could establish whether the ship is at rest or moving relative to the shore. Of course, this would be easy to do by simply looking out. But if one were in a cabin without portholes, then rectilinear motion of the ship could not be detected.

Galileo’s relativity can be shown mathematically to demand that the equations of motion of bodies, i.e., the equations of mechanics, must be identical in all the so-called inertial reference frames. These are coordinate systems attached to bodies moving uniformly along straight lines relative to very distant stars. (Obviously, in the case of Galileo’s ship, we disregard the diurnal rotation of the Earth, its rotation around the Sun, and the rotation of the Sun around the center of our Galaxy.)

Einstein’s extremely important achievement was that he extended Galileo’s relativity principle to all physical phenomena, including electrical and optical processes involving photons. This generalization of Galileo’s principle has resulted in drastic changes in fundamental ideas, such as those of space, time, mass, momentum, and energy. For example, in Einstein’s relativity the concepts of total energy and rest energy are introduced. The kinetic energy $T$ is related to total energy $E$ by

$$E = E_0 + T$$

where $E_0$ is the rest energy, related to the mass $m$ of a body by the famous formula

$$E_0 = mc^2.$$ 

The mass of photon being zero, its rest energy $E_0$ is also zero. For a photon, “a rest is but a dream”: it always moves with the velocity of light.
c. Other particles, e.g., electrons and nucleons, have nonzero masses and
therefore nonzero rest energy.

The relations between energy, velocity, and momentum for particles
with \( m \neq 0 \) take the following forms in Einstein’s mechanics:

\[
E = \frac{mc^2}{\sqrt{1-v^2/c^2}}, \quad p = Ev/c^2.
\]

Consequently,

\[
m^2c^4 = E^2 - p^2c^2.
\]

Each of the two terms on the right-hand side of the latter equation
increases as the body travels faster, but the difference remains constant
(physicists prefer to say that it is invariant). The mass of a body is a
relativistic invariant, since it is independent of the reference frame in
which the motion of the body is described.

Einstein’s relativistic formulas for momentum and energy become iden-
tical with the non-relativistic Newtonian expressions when \( v/c \ll 1 \).
Indeed, if we expand the right-hand side of the relation
\( E = mc^2/\sqrt{1-v^2/c^2} \) in a series in powers of the small parameter \( v^2/c^2 \), we
obtain the expression

\[
E = mc^2 \left[ 1 + \frac{1}{2} \frac{v^2}{c^2} + \frac{3}{8} \left( \frac{v^2}{c^2} \right)^2 + \ldots \right],
\]

where the dots represent higher powers of \( v^2/c^2 \).

\[ \square \] When \( x \ll 1 \), a function \( f(x) \) can be expanded in a series in powers
of the small parameter \( x \). By differentiating the left- and right-hand
sides of the expression

\[
f(x) = f(0) + xf'(0) + \frac{x^2}{2!} f''(0) + \frac{x^3}{3!} f'''(0) + \ldots,
\]

and each time evaluating the result for \( x = 0 \), you will readily verify
its validity (when \( x \ll 1 \), the discarded terms are negligible). Thus,

\[
f(x) = (1 - x)^{-1/2}, \quad f(0) = 1,
\]
\[
f'(x) = \frac{1}{2} (1 - x)^{-3/2}, \quad f'(0) = \frac{1}{2},
\]
\[
f''(x) = \frac{1}{4} (1 - x)^{-5/2}, \quad f''(0) = \frac{1}{4}. \square
\]

Note that for the Earth moving in its orbit with a velocity of 30 km/s,
the ratio \( v^2/c^2 \) is \( 10^{-8} \). For an air-liner flying at 1000 km/h, this
parameter is much smaller: \( v^2/c^2 \approx 10^{-12} \). Consequently, the
nonrelativistic relations

\[ T = \frac{1}{2} m v^2, \quad p = m v \]

are valid for the airliner to an accuracy of about \(10^{-12}\), and the relativistic corrections are definitely negligible.

Let us now return to the formula that relates mass, energy, and momentum, rewriting it in the form

\[ m^2 c^2 = (E/c)^2 - p_x^2 - p_y^2 - p_z^2. \]

The fact that the left-hand side of this equality is unaffected by going from one inertial reference frame to another is similar to the invariance of the square of momentum

\[ p^2 = p_x^2 + p_y^2 + p_z^2, \]

or of any squared three-dimensional vector, under rotations of the coordinate system (see above, Figure 1) in ordinary Euclidean space. In terms of this analogy, \(m^2 c^2\) is said to be the square of a four-dimensional vector, namely the four-dimensional momentum \(p_\mu\), where the subscript \(\mu\) takes on the four values \(\mu = 0, 1, 2, 3\) and \(p_0 = E/c, p_1 = p_x, p_2 = p_y, p_3 = p_z.\) The space in which the vector \(p_\mu = (p_0, \mathbf{p})\) has been defined is said to be pseudo-Euclidean. The prefix "pseudo" signifies in this case that the invariant is not the sum of the squares of all four components but is the expression

\[ p_0^2 - p_1^2 - p_2^2 - p_3^2. \]

The transformation that relates the time and space coordinates in two different inertial systems is called the Lorentz transformation. Without writing out this transformation in full, we note that if two events are separated by \(t\) in time and \(r\) in space, the quantity

\[ s = (ct)^2 - r^2, \]

called the interval, remains constant under the Lorentz transformation, i.e., it is a Lorentz invariant. Note that neither \(t\) nor \(r\) are invariants individually. When \(s < 0\), the interval is said to be timelike; when \(s > 0\), it is spacelike; when \(s = 0\), it is lightlike. When \(s < 0\), two spatially distinct events can be simultaneous in one reference frame but non-simultaneous in another.

Let us now consider a system of \(n\) noninteracting free particles. Let \(E_i\) be the energy of the \(i\)-th particle, \(p_i\) be its momentum, and \(m_i\) its mass.
The total energy and momentum of the system are, respectively,

\[ E = \sum_{i=1}^{n} E_i \quad \text{and} \quad p = \sum_{i=1}^{n} p_i. \]

Since, by definition, the mass is given by

\[ M^2 = E^2/c^4 - p^2/c^2, \]

the mass of the system is not, in general, equal to the sum of the masses of its constituents.

In our daily life, we are used to the equality \( M = \sum_{i=1}^{n} m_i \), but this fails for fast particles. For example, the combined mass of two electrons colliding head-on with equal energies is \( 2E/c^2 \), where \( E \) is the energy of each electron. In accelerator experiments, this combined mass is greater by many orders of magnitude than the quantity \( 2m_e \), where \( m_e \) is the mass of the electron.

We conclude this section with a few remarks about terminology.

Some books and popular-science periodicals use the phrases "rest mass" and "motional mass" (or the equivalent phrase "relativistic mass"). The latter mass increases with increasing velocity of the body. The "rest mass" \( m_0 \) is then taken to mean the physical quantity that we have simply called mass and denoted \( m \), and the relativistic mass \( m \) is taken to mean the energy of the body divided by the square of the velocity of light, \( m = E/c^2 \), which definitely increases with increasing velocity. This obsolete and essentially inadequate terminology was widespread at the beginning of this century when it seemed desirable, for purely psychological reasons, to retain the Newtonian relation between momentum, mass, and velocity: \( p = mv \). However, as we approach the end of the century, this terminology has become archaic and only obscures the meaning of relativistic mechanics for those who have not mastered its foundations sufficiently well.

In relativistic mechanics, the "rest mass" is neither the inertial mass (i.e., the proportionality coefficient between force and acceleration) nor the gravitational mass (i.e., the proportionality coefficient between the gravitational field and the gravitational force acting on a body). It must be emphasized that the infelicitous "motional mass" cannot be interpreted in this way, either.

The correct relation between the force \( F \) and the acceleration \( dv/dt \) can be found if we use the relativistic expression for the momentum.
\[ p = \frac{mv}{\sqrt{1 - \frac{v^2}{c^2}}}, \]
and recall that \( F = \frac{dp}{dt} \). The formula \( F = ma \), which is familiar from school textbooks, is valid only in the non-relativistic limit in which \( v/c \ll 1 \).

That the gravitational attraction is not determined by the "rest mass" is evident, for example, from the fact that the photon is deflected by a gravitational field despite its zero "rest mass". The gravitational attraction exerted on the photon by, say, the Sun, increases with increasing photon energy. We are therefore tempted to say that the phrase "motional mass" is meaningful at least in this case. In fact, this is not so. A consistent theory of the motion of a photon (or any object moving with velocity comparable with the velocity of light) in a gravitational field will show that the energy of a body is not equivalent to its gravitational mass.

To conclude this discussion of "mass", I must ask the reader never to use the phrases "rest mass" or "motional mass" and always to mean by "mass" the relativistically invariant mass of Einstein's mechanics.

Another example of unfortunate terminology is the false claim that high-energy physics and nuclear physics are somehow able to transform energy into matter and matter into energy. We have already pointed out that energy is conserved. Energy does not transform into anything, and it is only different particles that transform into one another. We shall discuss many examples of such transformation in the following pages. The point is well illustrated by the chemical reaction between carbon and oxygen that we observe in a bonfire. This reaction is

\[ C + O_2 \rightarrow CO_2 + \text{photons}. \]

The kinetic energy of photons and CO\(_2\) molecules is produced in this reaction because the combined mass of the C atom and the O\(_2\) molecule is slightly greater than the mass of the CO\(_2\) molecule. This means that while all the energy of the initial ingredients of the reaction is in the form of rest energy, that of the final products is the sum of rest energy and kinetic energy.

Energy is thus conserved but the carriers of it, and the form in which it appears, do change.